Linearization of the isotropy subalgebra of a transitive Lie algebra of vector fields

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Let $\mathcal{A}$ be a finite-dimensional Lie algebra of vector field germs at $0 \in \mathbb{R}^n$ which contains vector fields $\partial/\partial x_i + h.o.t,\ i = 1, ..., n$ (such algebras are called transitive) and let $\mathcal{I} = \{V \in \mathcal{A} : V(0) = 0\}$ be the isotropy subalgebra of $\mathcal{A}$. The linear approximations at 0 of the vector fields of $\mathcal{I}$ form a Lie algebra $j^1\mathcal{I}$. Assume that $dim\,\mathcal{I} = dim\,j^1\mathcal{I}$ so that $j^1\mathcal{I}$ is a faithful representation of $\mathcal{I}$ in $gl(n)$. Under which condition $\mathcal{I}$ and $j^1\mathcal{I}$ are diffeomorphic, i.e. can be sent one to the other by a local diffeomorphism of $\mathbb{R}^n$? I will discuss this question from various points of view and will formulate and explain some unexpected theorems, for example that $\mathcal{I}$ and $j^1\mathcal{I}$ are diffeomorphic if $dim\,\mathcal{I} = dim\,j^1\mathcal{I} = 1$, without any restrictions on the eigenvalues of vector field which span $\mathcal{I}$.