Geometric methods in the N-body problem

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Abstract

We consider a modified N-body problem with potential of the form $1/r^2$ on the level of energy $h = 0$ and angular-momentum equal to zero. Once the symmetries are reduced, the dynamics of solutions is equivalent to a geodesic flow on the Riemannian manifold $\mathbb{CP}^{N-2}$ minus the compact set of the collisions. The metrics is the Jacobi-Maupertuis metric on the fixed energy level.

For $N = 3$, we study the geodesic flow in a two dimensional conformal shape sphere when the masses are equal. Here, it is well-know that the corresponding metric is hyperbolic except in the points of Lagrange, where the Gaussian curvature is zero. This led to deep dynamical consequences, see [2]. On the other hand, J. Connor and R. Montgomery in [1] proved that the negative curvature property does not persist for $N = 4$. In this talk we deal with these problems and the 5-body problem. In particular we show numerically that the reduced 5-body configuration space has positive sectional curvature for some planes. This is a joint work with J. Antonio García, Josué Meléndez and J. Guadalupe Reyes Victoria.

References

[1] C. Jackman and R. Montgomery. *No hyperbolic pants for the 4-body problem with strong potential*, Pacific J. Math. 280 (2016), 401–410.

[2] R. Montgomery. *Fitting hyperbolic pants to a three-body problem*, Ergodic Theory Dynam. Systems 25 (2005), 921–947.