GEOMETRIC, ALGEBRAIC, AND ANALYTIC PROPERTIES OF FIGURE-EIGHT SOLUTION

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In the morning of June 14, 2001, I found a short column on a newspaper, which read "New solution in three-body problem, figure-eight solution, was found". I was really astonished and attracted by this solution. So, I made a movie of the figure-eight solution, and looked on moving three bodies and the orbit joyfully. After a couple of hours (or days, I'm not sure now), I suddenly realized that three tangent lines at three bodies meet at a point for each instant. This discovery made me to start investigating properties of figure-eight solution.

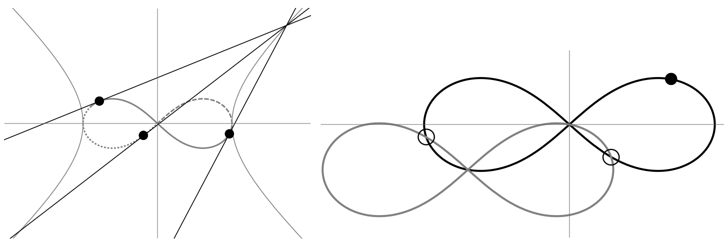

Figure 1. Left: Three tangents theorem, three tangent lines of the figure-eight orbit at the three bodies meet at a point. Right: Three points theorem. For given one point (solid circle) on the figure-eight orbit (black), the other two points (hollow circle) are found in the crossing points of the figure-eight orbit (black) and its translational copy (gray).

In my talk, I will summarize the properties of the figure-eight solution, that I and my collaborators found. That will includes "three tangents theorem", "three points theorem", "convexity of each leaf", "synchronized similar triangles", "solution on the lemniscate". Recently, I started new projects with my colleagues. I hope I will have some clear results by the meeting, and can talk new stories of the figure-eight solution.

In the following URL, you will find my papers, figures and movies for the figure-eight solution. http://www.clas.kitasato-u.ac.jp/~fujiwara/nBody/nbody.html

Question: Take three points $(x_i, y_i) \neq (0,0)$, $\sum x_i = \sum y_i = 0$, $i = 1,2,3$, on the lemniscate $(x^2+y^2)^2 = x^2 - y^2$. Then, we have $\sum(x_i^2 + y_i^2) = \sqrt{3}$, and three tangent lines of the lemniscate at the three points must meet at a point on the rectangular hyperbola $x^2 - y^2 = 1$. To prove these properties, we have used an explicit parametrization of (x_i, y_i) by Jacobian elliptic function. However, the "three point theorem" says that the above conditions uniquely determine the two points for given one point. So, we 'know' that we don't need explicit parametrization for proof. Can anyone prove these properties only by algebraic calculations? I don't know how. I also don't know whether this is a just simple 'aha' problem or a hard problem. No prize money `(^_^)`.